

# Modelling flow-induced vibrations of tandem cylinders in the post-critical flow regime

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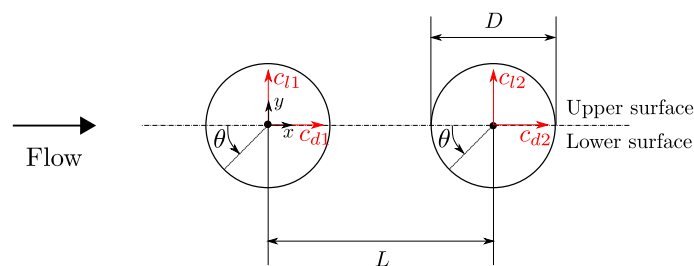
## SUMMARY:

This work focuses on the mathematical modelling of the flow-induced vibrations of tandem cylinders. The post-critical flow regime characterised by a fully turbulent state is investigated. This type of phenomena can take place on large structures submitted to strong wind and leading to Reynolds number above  $10^6$ . In particular, tandem arrangement characterised by closely spaced cylinders ( $L/D = 1.2$ ). On the basis of experimental aeroelastic tests, the specific case of VIV-galloping interaction is considered. The proposed model consists in coupled structural and wake oscillators combined with quasi-steady aerodynamic force coefficients. The Tamura wake-oscillator modelling the dynamics of the wake lamina is selected and a single wake is considered. A time delay parameter will be introduced in the quasi-steady forces. Unsteady pressure fields measured on static cylinders are used to identify the empirical coefficients of the model. The outputs of the model will be compared to aeroelastic responses measured in the wind tunnel for validation purpose.

*Keywords: Tandem cylinders, mathematical modelling, VIV-galloping interaction*

## 1. CONTEXT

Cylinder-like structures can be found in several engineering applications. In some cases, a flow field develops around them and may lead to fluid-structure phenomena, such as Vortex-Induced Vibrations (VIV) or galloping. In this research, the arrangement of interest consists in tandem cylinders, as shown in Fig. 1. This arrangement corresponds to different applications: twin chimneys, twin towers or the Hyperloop concept, among others. Such structures are characterised by large dimensions which lead to a flow in the post-critical regime, i.e. the boundary layers, the shear layers and the wakes are fully turbulent. Several experimental works exist in the literature but only a few studies focused on the modelling aspects of the flow-induced vibrations of tandem cylinders.



**Figure 1.** Tandem cylinders in a crossflow.

Ruscheweyh (1983) investigated the interference galloping by considering the displacement of the rear cylinder only. The concept of hysteresis lag angle between the motion of the cylinder and the aeroelastic force acting on it was introduced. Based on the quasi-steady theory, he retrieved an expression to predict the onset flow velocity of the instability which depends on the square root of the mass-damping parameter, the square root of the spacing ratio and an interference galloping criterion (including the hysteresis lag angle). Paidoussis and Price (1989) focused on the flow-induced instabilities of arrays of cylinders in a crossflow. They also made use of a time delay parameter which is similar to the hysteresis lag angle in Ruscheweyh (1983). Hémon (1999) improved the concept of the time delay with physical interpretations. His work was done for a relatively large spacing ratio ( $L/D = 3$ ). Recently, Fan et al. (2021) developed a model dealing with VIV-galloping interaction of closely spaced tandem cylinders. It consists in coupled structural and wake oscillators and is based on the work of Mannini et al. (2018).

The purpose of this work is to investigate the existing mathematical models and combine them into a new one with physical interpretations for the different parameters. It is based on experimental data gathered during a wind tunnel test campaign performed on a static and dynamic tandem cylinders at the Wind Tunnel Lab of University of Liège. The cylinders are considered in close proximity: a centre-to-centre spacing ratio  $L/D$  of 1.2.

## 2. METHODOLOGY

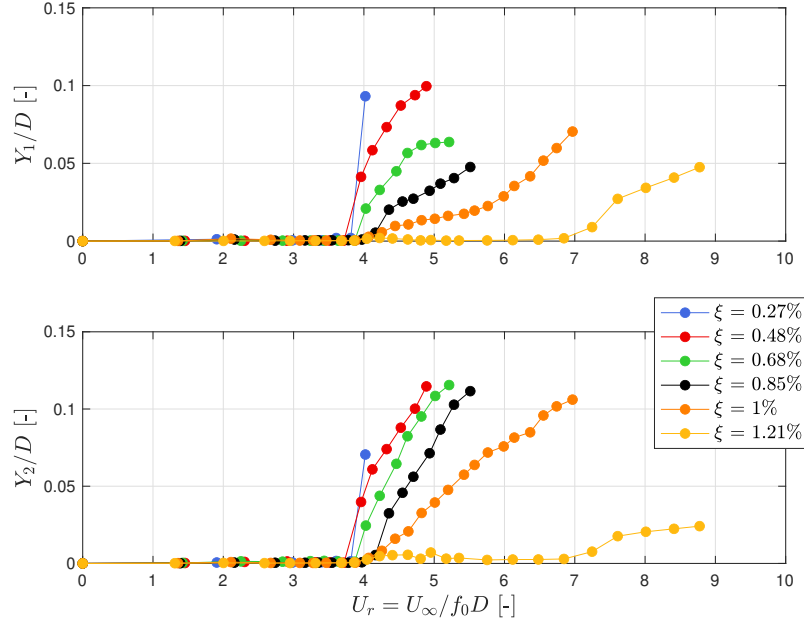
As a starting point, the unsteady pressure fields were measured around a static cylinders (see Fig. 2(a)) for different wind incidences ranging from  $0^\circ$  to  $10^\circ$ . The complete experimental details can be found in Dubois and Andrienne (2022). Those static measurements are used as empirical data to identify the values of the different parameters in the mathematical model.

Then, aeroelastic tests were performed (see Fig. 2(b)). They consisted in measuring the displacement of each cylinder and the velocity in the wake while varying the wind velocity and the structural damping ratio. Those measurements are intended for validation of the mathematical model. In Fig. 2, it can be observed that surface roughness was added on the cylinders to trigger the post-critical flow regime at lower Reynolds numbers.



**Figure 2.** Experimental models in the test section: static (left), dynamic (right).

Fig. 3 shows the variation of the amplitude of vibration of each cylinder with the reduced velocity for different structural damping ratios gathered during the wind tunnel test campaign. For each damping ratio, the flow-induced vibrations start at a reduced velocity equal to 4. This value corresponds to the critical reduced velocity of VIV ( $U_r^{VIV}=1/St$ ) associated with a Strouhal number of 0.25. Then, the amplitudes of vibration always increase with the reduced velocity for damping ratio lower than 1.21%. For a damping ratio of 1.21%, a clear decoupling between the VIV and galloping phenomena can be observed. Based on this observation, it can be stated that VIV-galloping interaction occurs for lower values of the damping ratio.



**Figure 3.** Variation of the amplitudes of vibration with the reduced velocity for different damping ratios.

Therefore, the new mathematical model must be able to deal with the VIV-galloping interaction, similarly to what has been done by Mannini et al. (2018). The model developed by Fan et al. (2021) is a starting point and the objective is to improve it with physical interpretation based on the experimental data. One degree of freedom in the transverse direction of the flow is considered for each cylinder. The wake is modelled by a single degree of freedom because of the close proximity of the cylinders and the resulting small gap.

The generic equations of the proposed model are:

$$\ddot{Y}_1 + 2\xi\dot{Y}_1 + Y_1 = s_0 U_r^2 \left[ a_1 \alpha - b_1 \frac{\dot{Y}_1}{U_r} + C_{F_{y1}}^{QS}(Y_1, \dot{Y}_1, Y_2, \dot{Y}_2) \right], \quad (1)$$

$$\ddot{Y}_2 + 2\xi\dot{Y}_2 + Y_2 = s_0 U_r^2 \left[ a_2 \alpha - b_2 \frac{\dot{Y}_2}{U_r} + C_{F_{y2}}^{QS}(Y_1, \dot{Y}_1, Y_2, \dot{Y}_2) \right], \quad (2)$$

$$\ddot{\alpha} - 2\beta v \left( 1 - \gamma \alpha^2 \right) \dot{\alpha} + v^2 \alpha = F(\dot{Y}_1, \dot{Y}_2, \ddot{Y}_1, \ddot{Y}_2, U_r), \quad (3)$$

where the operator  $\dot{x} = \partial x / \partial \tau$  with  $\tau = \omega t$ ,  $Y_i = y_i / D$  is the dimensionless displacement of  $i^{th}$  cylinder,  $\alpha$  is the angle characterising the dynamics of the wake,  $\xi$  is the structural damping ratio,  $s_0$  is a dimensionless constant depending on the mass ratio,  $U_r = U_\infty / f_0 D$  is the reduced velocity and  $v = U_r St$ .

Eqs. (1)-(2) correspond to structural oscillators. On the right side of the structural equations,  $C_{F_{y_i}}^{QS}$  is the quasi-steady aerodynamic force coefficient acting on the  $i^{th}$  cylinder and depends on the displacement and velocity of each cylinder. This aerodynamic coefficient is extracted for each cylinder from the experiments performed on the static set-up. A time delay parameter will be identified in this coefficient with respect to the displacement of the cylinders. Then, the coefficients  $a_i$  and  $b_i$  are respectively linked to the vortex-induced excitation and aerodynamic damping on the  $i^{th}$  cylinder.

In Eq. (3) describing the wake dynamics,  $\beta$  and  $\gamma$  are coefficients related to the dimensions of the wake lamina. On the right side,  $F$  is a function describing the effect of the cylinders' motion on the wake dynamics. The different coefficients ( $a_i$ ,  $b_i$ ,  $\beta$ ,  $\gamma$ ) and function ( $F$ ) will be identified from the unsteady pressure fields measured on static cylinders.

### 3. CONCLUSIONS

This work aims at developing a new mathematical model of flow-induced vibrations of tandem cylinders in the post-critical flow regime. The model will consist in coupled structural and wake oscillators combined with quasi-steady aerodynamic force coefficients in order to deal with the VIV-galloping interaction. The latter interaction was observed during a wind tunnel test campaign on an aeroelastic model. Unsteady pressure fields around the static cylinders were measured and they will be used as input data to the mathematical model. Finally, the model will be validated by comparison with aeroelastic experimental responses.

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